**INPUTS AND COSTS**

Production is the process of turning inputs into outputs and the analysis of production and production function is important because it defines the cost function, an important determinant of optimal production: a specific production function is associated to a specific cost.

A production function is a mathematical function linking the inputs used by a firm in the production process and the quantity of output it produces. A **fixed input**is an input whose quantity is fixed for a period and cannot be varied (land, costly machinery, etc.). These inputs do not depend on the quantity of output produced. A **variable input**is an input whose quantity the firm can vary at any time and depends on the quantity of output that the firm wants to produce (labour force in flexible labour markets).

The most common production function is the Cobb-Douglas one which depends on labour (L) and capital (K):

A represents the general level of technology available to the firm, whereas α and β are positive constants which define respectively how much labour and how much capital contributes to the final output. The value of their sum has important consequences in terms of returns to scale of a firm.

In the short run, the distinction between fixed and variable inputs makes sense because we assume that in the short run the firm cannot change the quantity of a fixed input, whereas in the long run the firm has more flexibility to modify the quantity of land and the size of buildings and machinery. The long run per se is the period in which all inputs can be defined as variables because the firm can change also land, machinery and buildings, whereas the short run is the period in which at least one input can be considered as fixed.

The total product curve shows the quantity of output that can be produced using certain quantities of variable inputs keeping a given quantity of at least one fixed input.

e.g. considering a fixed input (land, K = 10) and a variable one (L), we have a table in which we report the quantity of labour and we want to understand how much output can be produced for every number of workers (L).

When we increase the number of labour, also the quantity of output increases. The total product curve represents the relationship between the variable input and output keeping constant the fixed input. The third column shows the marginal product of labour. The **relationship** is **positive** but **not constant**: marginal product of labour changes along the production function, since it is initially very large but declines along the total product curve. So, when the variable input is used in small quantities the marginal product of labour is very high, yet it declines when the variable input is increased, so we observe a decreasing marginal product of labour.

**Marginal product** is the **change in output resulting from a one-unit increase in the amount of labour input (ΔQ/ΔL)**. We can define it as the ratio between the difference in output over difference in variable inputs used in the production process. Marginal product initially rises as more workers are hired; then it declines. In general, there are **diminishing** returns to an input when an increase in the quantity of that input, holding the quantity of all other inputs fixed, reduces that input’s marginal product. Due to diminishing returns to labour, the **MPL curve** is **negatively sloped**.

In the graph on the left, the relationship between inputs used and output produced is positive, yet it is concave, so the slope is positive but the second derivative is negative. In the graph on the right, MPL is represented as a negative curve because if we add additional workers the marginal output they are able to produce decreases. This is because, having a certain amount of fixed inputs, even if we keep adding a number of labour the quantity cannot increase at infinity, because the land is occupied by too many workers and the additional output for each of them is reduced.

What happens to the total product curve and to the marginal product curve if the same firm decides to **increase** the quantity of land, namely the use of **fixed input**, doubling the quantity from 10 to 20?

The **total product curve** will **shift upwards**: each worker will produce more, gaining profit from the availability of more fixed input. Also the **marginal product curve** will shift upwards, so the relationship between variable input, fixed input and the two curves is positive but not constant.

There are costs associated to inputs which define the optimal production decision and the optimal supply curve of a firm: a **fixed cost**is a cost that does **not depend** directly on the **quantity of output** that the firm intends to produce. It is the cost associated to the fixed input and it is expressed by the function **FC(Q)**. A **variable cost**is a cost that **depends** on the **quantity of output** that the firm intends to produce. It is the cost of the variable input and it is expressed by the function **VC(Q)**. The **total cost function** is the sum of fixed and variable costs, expressed by  
**TC(Q ) = FC(Q) + VC(Q)**.

The **total** **cost curve** becomes steeper as more output is produced as a result of diminishing returns or as a result of a decreasing marginal product of labour. The curve is **convex** (both first and second derivative are positive), so TC increases as Q increases and the marginal cost of producing additional quantities of output is higher for higher levels of production. The curve has this shape because of diminishing marginal product of labour: additional workers will increase the costs of production, so the curve has an **exponential shape**.

This is a **static representation** of the production process, assuming that in a given range of time there is a constant relationship between inputs and outputs.

**MARGINAL COST**

The **marginal cost** is the **change in total cost generated by one additional unit of output**.It is the ratio of the change in total cost for a firm over the total change in output produced. Q is a continuous variable, divisible in infinite points. The marginal cost is the derivative of the total cost function with respect to Q.

**MC = ΔTC/ΔQ**

The marginal cost curve is upward sloping because there are diminishing returns to inputs. As output increases, the marginal product of the variable input declines. This implies that more and more of the variable input must be used to produce each additional unit of output as the amount of output already produced rises. And since each unit of the variable input must be paid for, the cost per additional unit of output also rises.

**AVERAGE COST**

The **average** **variable cost** is the variable cost per unit of output produced and is defined by **AVC = VC/Q**

The **average** **fixed** **cost** is the fixed cost per unit of output produced and is defined by **AFC = FC/Q**

The **average total cost** (often referred to as average cost in general) is the total cost per unit of output produced and is defined by **ATC = TC/Q** or **ATC = AVC + AFC**

Increasing output (Q) has **two** opposing **effects** on average total cost:

* **Spreading effect:** reduction in AFC which causes a reduction in ATC. In fact, the larger the output, the more output over which fixed cost is spread, leading to lower average fixed cost. In fact, if Q increases TC will be divided in a higher number and the result will be lower.
* **Diminishing returns effect:** the larger the output, the more variable input required to produce additional units, which leads to higher average variable cost.

In this average total cost curve, we can observe that as Q increases ATC initially decreases and then starts increasing. In fact, the first decreasing part represents the spreading effect overcoming the diminishing returns effect and leading to a downward slope, while the second increasing part represents the diminishing returns effect overcoming the spreading effect and leading to an upward slope.

The **average variable cost curve** is always **below** the **average total cost curve**, which starts decreasing (short run) and continues increasing (long run). In fact, the ATC curve comprehends also the AFC, so AVC is only a component. Moreover, if graphically the marginal cost curve is below the ATC curve, the ATC has a negative slope and vice versa: in fact, if an additional unit costs less than ATC, the average cost will decrease and vice versa. The marginal cost curve intersects the average total cost curve only in its minimum, which is the only point in which it does not change (neither positive nor negative slope).

To sum up:

* **Marginal cost** is **upward sloping** because of diminishing returns.
* **Average variable cost** also is **upward sloping** but is **flatter** than the marginal cost curve.
* **Average fixed cost** is **downward sloping** because of the spreading effect.
* The **marginal cost curve intersects** the **average total cost curve** from below, crossing it at its **lowest point**.

The same kind of analysis can be done with slightly different curves, such as **U-shaped** **AVC** curves, more frequent in reality: in this case, the marginal cost curve still intersects the average total cost at its minimum, but also the average variable cost curve at its minimum. The shape of the latter depends on the total cost function.

In the analysis of production, it is important to consider the difference between increasing, constant and decreasing return to scale.

* **Increasing returns to scale (economies of scale):** long-run average total cost declines as output increases.
* **Decreasing returns to scale (diseconomies of scale):** long-run average total cost increases as output increases.
* **Constant returns to scale:** long-run average total cost is constant as output increases.

These three definitions are particularly relevant in monopoly and are defined by the production function.

Both inputs are variable in the long run, so all costs vary, including the fixed ones. In the short run, some costs are fixed and some are variable. The firm will choose its fixed cost in the long run based on the level of output it expects to produce.

Considering the long run, there is a trade-off between the amount of variable cost and fixed cost the firm wants to cover and it depends on the total output it wants to produce. So there is a trade-off between higher fixed cost and lower variable cost for any given output level and vice versa.

If the company wants to produce a small amount of output, it is better to maintain a low fixed cost because the ATC will be maintained low. If the firm wants to produce more, it will have an incentive to increase the amount of fixed costs (e.g. a bigger building) in the long run, because it will be more distributed and the ATC will be lower. The decision of the firm in the long run depends on the quantity of output it intends to produce.

What is the relationship between the short run and long run ATC curve? The long-run average total cost curve shows the relationship between output and average total cost when fixed cost has been chosen to minimize average total cost for each level of output.

Assuming that at the beginning the firm wants to produce 6 units, in the short run it produces 6 units of output minimizing its cost. If the firm wants to reduce the amount of output to 3 units in the short run, it will have no way to modify the fixed costs and therefore the ATC will increase. In the long run, it will probably reduce the fixed inputs minimizing the ATC. If the firm wants to increase output, the ATC will increase as well. The long run curve touches all the short run curves in their minimum points.

In the long run, the return to scale is increasing (economies of scale), namely the ATC in the long run decreases when we increase the amount of output. It is typical of industries in which there is a small number of companies working and therefore large share on the market and low competition. It is better to have few companies because increasing output leads to a decreasing ATC. The return to scale is constant when long-run ATC is constant with increased output. This is typical of small companies, where output increases or decreases proportionally to the amount of inputs. The return to scale is decreasing (diseconomies of scale) when long-run ATC increases as output increases.